April 2020

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Acceleration field

→ Fluid motion can be described by
→ Langrangian
or → Eulerian

In either case
applying $F = ma$,
we can obtain particle acceleration

→ in Langrangian method
→ fluid acceleration is described just as in solid body dynamics

$a = a(t)$ for each particle

→ in Eulerian method
→ acceleration is described just as a function of position and time without following the particle
→ it is analogous to describing the flow in terms of velocity field $V = V(x, y, z, t)$ rather than the velocity of particular particles

→ Consider a fluid particle moving along a pathline

particle velocity (V_A)
is a function of its location and time

$$V_A = V_A\left[x_A(t),\ y_A(t),\ z_A(t),\ t\right]$$

acceleration

$$\vec{a}_A(t) = \frac{d\vec{V}_A}{dt}$$

Applying chain rule

$$= \frac{\partial \vec{V}_A}{\partial t} + \frac{\partial \vec{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \vec{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \vec{V}_A}{\partial z}\frac{dz_A}{dt}$$

$$u_A = \frac{dx_A}{dt}$$

$$v_A = \frac{dy_A}{dt}$$

$$w_A = \frac{dz_A}{dt}$$

$$\vec{a}_A = \frac{\partial \vec{V}_A}{\partial t} + u_A \frac{\partial \vec{V}_A}{\partial x} + v_A \frac{\partial \vec{V}_A}{\partial y} + w_A \frac{\partial \vec{V}_A}{\partial z}$$

This is valid for any particle

$$a = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Scalar components

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

a_x, a_y, a_z are x, y, z components

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$$\vec{a} = \frac{D\vec{V}}{Dt}$$

$$\frac{D}{Dt}(\ ) = \frac{\partial}{\partial t}(\ ) + u\frac{\partial}{\partial x}(\ ) + v\frac{\partial}{\partial y}(\ ) + w\frac{\partial}{\partial z}(\ )$$

Material / or Substantial Derivative

$$\boxed{\frac{D}{Dt}(\ ) = \frac{\partial}{\partial t}(\ ) + (\vec{V}\cdot\nabla)(\ )}$$

Short form

$$\vec{V}\cdot\nabla = u\frac{\partial}{\partial x}(\ ) + v\frac{\partial}{\partial y}(\ ) + w\frac{\partial}{\partial z}(\ )$$

For example

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$= \frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T$$

Temperature

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→ Time derivative, $\frac{\partial}{\partial t}(\ )$ local derivative

→ Spatial derivative, $\frac{\partial}{\partial x}(\ )$, $\frac{\partial}{\partial y}(\ )$, $\frac{\partial}{\partial z}(\ )$

Material derivative contains two types of terms

→ Time derivative (local)
→ Spatial derivative

local derivative represents
→ unsteadiness of the flow

$\frac{\partial V}{\partial t}$ — local acceleration

$\frac{\partial}{\partial t}(\ ) = 0$ — steady flow

→ Physically there is no change in flow parameters at a fixed point in space if the flow is steady

→ there may be spatial variations for (velocity, temperature, density)

Unsteady $\frac{\partial V}{\partial t} \neq 0$,

$\frac{\partial T}{\partial t} \neq 0$

$\frac{\partial \rho}{\partial t} \neq 0$

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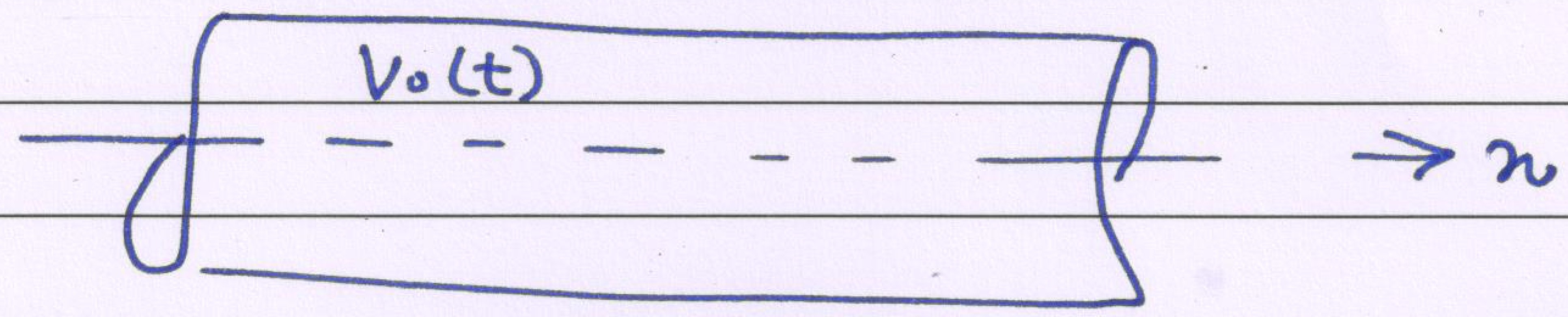
Uniform flow

→ When velocity & other hydrodynamic parameters at any instant of time do not change from point to point in a flow

→ flow is said to be uniform
→ flow can be spatially uniform in a pipe

Consider flow thru a constant diameter pipe

$$\vec{V} = V_0(t)\hat{i}$$



→ Value of acceleration depends on whether V_0 is being increased or decreased

$$\frac{\partial V_0}{\partial t} > 0 \qquad \frac{\partial V_0}{\partial t} < 0$$

— steady uniform flow
— unsteady uniform flow
— unsteady non-uniform flow
— steady non-uniform flow

Various types

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for a uniform flow (as shown in figure)

$u\frac{\partial u}{\partial x}$, $v\frac{\partial v}{\partial y}$, etc vanish

$\frac{\partial u}{\partial x}$, $v, w = 0$

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\cancel{\frac{\partial \vec{V}}{\partial x}}^{0} + v\cancel{\frac{\partial \vec{V}}{\partial y}}^{0} + w\cancel{\frac{\partial \vec{V}}{\partial z}}^{0}$$

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} = \frac{\partial V_0\,\hat{i}}{\partial t}$$

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Convective Effects

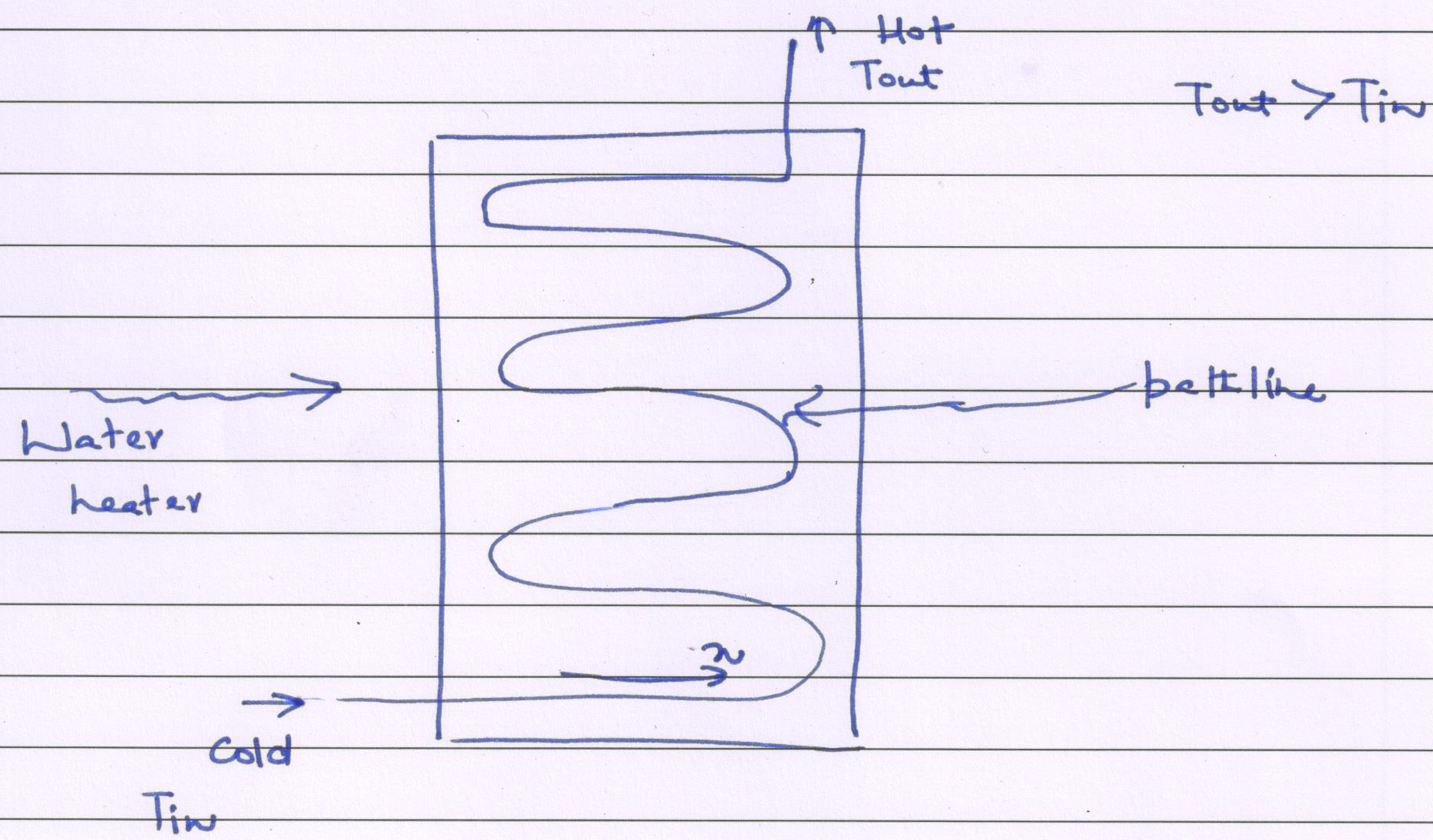
— portion of the material derivative represented by the spatial derivative — convective derivative

→ flow property associated with a fluid particle may vary from point to point

$$(\vec{V} \cdot \nabla)\vec{V} \leftarrow \text{Convective acceleration}$$

Example:

Steady state operation of water heater



→ Water entering the heater is always at same cold temperature

→ Water coming out from the heater is always at the same hot temperature

→ flow is steady

→ However, T of water increases as it passes thru the heater

$$T_{out} > T_{in}$$

$$\frac{DT}{Dt} \neq 0$$

$$\frac{\partial T}{\partial t} = 0 \quad [\text{steady}]$$

But $u\frac{\partial T}{\partial x} \neq 0$

(x — being along the streamline)

T increases with x

→ There is non-zero temperature gradient along the streamline

→ a fluid traveling along this non-constant temperature path $\left(\frac{\partial T}{\partial x} \neq 0\right)$ at a specified speed (u) will have a temperature change with time at a rate

$$\frac{DT}{Dt} = u\frac{\partial T}{\partial x}$$

even though the flow is steady (i.e. $\frac{\partial T}{\partial t} = 0$)